Government College of Engineering and Research, Avasari(Khurd)

Department: Mechanical Engineering

Learning Resource Material (LRM)

Name of the course: Mechanical System Design **Course Code:** 402048

Name of the faculty: J. M. Arackal **Class:** BE(Mech)

SYLLABUS (Unit 1)

Unit 1: Design of Machine Tool Gearbox (8 Hours)

Introduction to machine tool gearboxes, design and its applications, basic considerations in design of drives, determination of variable speed range, graphical representation of speed and structure diagram, ray diagram, selection of optimum ray diagram, deviation diagram, difference between numbers of teeth of successive gears in a change gear box.

Lecture Plan format:

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Unit No	Lecture No.	Topics to be covered	Text/Reference Book/ Web Reference
		UNIT 1	
1	1	Introduction to machine tool gearboxes, design and its applications	1,2
1	2	Basic considerations in design of drives, determination of variable speed range	1,2
1	3	Graphical representation of speed and structure diagram, ray diagram	1,2
1	4	Problems on Formation of Structural Equation	1,2
1	5	Problems on Speed Diagram	1,2
1	6	Problems on Ray Diagram	1,2
1	7	Problems on Design Layout of gearing mechanism for a particular speed range	1,2
1	8	Problems on Design Layout of gearing mechanism for a particular speed range	1,2

List of Text Books /Reference Books/ Web Reference

1-Bhandari V.B. ―Design of Machine Elements‖, Tata McGraw Hill Pub. Co. Ltd.

2-R.K. Jain- Machine Design, Khanna Publishers

3-Johnson R.C., ―Mechanical Design Synthesis with Optimization Applications‖, Von Nostrand Reynold Pub

Design of Speed Box

- Used for stepped regulation of rpm.
- Between two extreme available values n_1 & n_z ... what should be criteria for choosing descrete steps [Let z be no of steps]

Module 1: Design of m/c tool gear boxes.

Laws of speed range distribution

a) RPM Values constitute Arithmetic progression

$n_1 = n$

$n_2 = n_1 + a = n + a$

$n_3 = n_2 + a = n_1 + 2a$.

$\therefore \boxed{n_z = n_1 + (z-1)\,a}$

we have $v = \dfrac{\pi d n}{1000}$

$\Rightarrow d = \dfrac{1000 v}{\pi n}$.

$\therefore$ upper limit of dia

$$d_x = \frac{1000\,v}{\pi\, n_x}.$$

Lower limit of dia

$$d_{x+1} = \frac{1000\,v}{\pi\, n_{x+1}}$$

$$\therefore \Delta d_x = \frac{1000 v}{\pi}\left[\frac{1}{n_x} - \frac{1}{n_{x+1}}\right]$$

Let $n_1 = 30$ $\quad n_z = 375$ $\quad z = 12$ $\quad v = 20\ m/min$.

n_x rpm.	ϕ	d_x	Δd_x
$n_1 =$ 30.	2.04.	212.18	108.48
$n_2 =$ 61.4	1.57.	103.7.	35.1
$n_3 =$ 92.8	1.33.	68.6	17.3
$n_4 =$ 124.2	1.25	51.25	10.34.
$n_5 =$ 155.6	1.21	40.91	6.87.
$n_6 =$ 187	1.17.	34.04	4.89.
$n_7 =$ 218.4	1.14.	29.15	3.67
$n_8 =$ 249.8	1.13.	25.48	2.84.
$n_9 =$ 281.2	1.11	22.64.	2.28.
$n_{10} =$ 312.6	1.10	20.36	1.86.
$n_{11} =$ 344	1.09.	18.50.	1.53.
$n_{12} =$ 375.		16.97.	

$a = \dfrac{n_z - n_1}{11} = \dfrac{375 - 30}{11} = \underline{\underline{31.4}}$

Besides the above ...

- At high rpm range some values of speed steps also redundant.
- Low range, need to add more steps between the calculated values.

b) RPM values constitute GP (Geometric Progression)

$n_1 = n$

$n_2 = \phi n_1$

$n_3 = \phi n_2 = \phi a n_1 = a^2 n_1$

$n_z = \phi^{z-1} n_1$

$\phi = \left(\frac{n_z}{n_1}\right)^{\frac{1}{z-1}} = 1.26.$

$v = \frac{\pi d n}{1000}$. $d = \frac{1000}{\pi n}$ $= \frac{1000 \times 20}{\pi n}$.

n_x	ϕ	d_x	Δd_x
$n_1 = 30$	1.26	212.18	43.78
$n_2 = 37.8$	1.26	168.4	34.76
$n_3 = 47.63$	1.26	133.64	27.58
$n_4 = 60.014$	1.26	106.06	21.88
$n_5 = 75.62$	1.26	84.18	17.38
$n_6 = 95.3$	1.26	66.8	13.67
$n_7 = 120$	1.26	53.04	10.95
$n_8 = 151.2$	1.26	42.09	8.68
$n_9 = 190.51$	1.26	33.41	6.89
$n_{10} = 240$	1.26	26.52	5.47
$n_{11} = 302.4$	1.26	21.05	4.15
$n_{12} = 375$	1.26	16.9	

effective than AP

ometric progression is commonly used in machine tool rives owing to the following advantage,

1) Constant loss of economic cutting speed.

suppose spinde rpm constitute the following series -

$$n_1\ n_2\ n_3\ ....\ n_j\ n_{j+1}$$

Let V_{opt} lie between n_j & n_{j+1}

$\therefore$ we have.

$$\frac{\pi d n_j}{1000} < V_{opt} < \frac{\pi d n_{j+1}}{1000}.$$

The difference between the actual cutting speed & the optimum cutting speed is known as the loss of economic cutting speed. the optimum cutting loss is maximum when the middle of two speeds speed lies in the

$$V_{opt} = \frac{\pi d}{1000}\left(\frac{n_j + n_{j+1}}{2}\right).$$

$$\therefore (\Delta V_f)_{max} = \frac{\pi d}{1000}\left(\frac{n_{j+1} - n_j}{2}\right).$$

$\therefore$ from the above we have.

$$(\Delta V_f)_{max} = \left(\frac{n_{j+1} - n_j}{n_{j+1} + n_j}\right) V_{opt}.$$

Denoting $\frac{n_{j+1}}{n_j} = \phi_j$ we have.

$$(\Delta V_f)_{max} = \left(\frac{\phi_j - 1}{\phi_j + 1}\right) V_{opt}.$$

So if $(\Delta V_f)_{max}$ is to remain constant.

ϕ_j should be constant.

ie it should be in G.P.

2) Better Design features.

If the rpm values are obtained by mounting new pair of gears on the shaft everytime, then changing of speeds becomes time-consuming. inconvinient & infeasible.

If the rpm values are obtained by mounting gears of appropriate transmission ratio on the shaft permanently, then axial dimensions of such a speed box becomes too large.

So it concludes that speed steps in a speed box should be obtained not through a single transmission between two shafts but through group of transmissions between number of shafts. this is possible if it is in GP(ϕ)

i) If members are so removed that only every x^{th} member remains, then the resulting series in GP having progression ratio ϕ^x.

ii) If ϕ is multiplied by a constant A, the resulting series is a GP having same progression ratio but whose members are A times greater.

iii) If ϕ is multiplied by ϕ^x, the resulting series is a GP. which is shifted by x members.

3rd) constant loss of productivity in whole rpm range.

Metal removed in unit time $= \pi d n . S = 1000 S V$.

$S \rightarrow$ feed, mm/rev.

for a constant value of, feed s & depth of cutt, the productivity of a machining operation is constant in the whole rpm range.

al information required for designing a
peed Box : Selection of Range ratio

The following information is required.

1) The highest output, n_{max}.
2) The lowest output, n_{min}.
3) The number of steps Z into which the range is divided.
4) no of stages in which the reqd number of steps are to be achieved

Range Ratio, $R_n = \frac{n_{max}}{n_{min}} = \frac{V_{max}\, d_{max}}{V_{min}\, d_{min}} = R_v R_d$.

fixing of higher speed limit involves productivity loss in some machining operation. A wide range is not practicable. R_v should be kept within reasonable limit.

Range of diameter should also be selected on the basis of statistical study of the working of similar machine tools.

$$R_n = \frac{n_{max}}{n_{min}} = \phi^{z-1}$$

$\frac{n_{max}}{n_{min}}$

$$\phi = (R_n)^{1/z-1}$$

$$\log \phi = \frac{1}{z-1} \log R_n$$

$$z-1 = \frac{\log R_n}{\log \phi}$$

$$z = \frac{\log R_n}{\log \phi} + 1 = \frac{\log R_n + \log \phi}{\log \phi}$$

$$z = \frac{\log R_n \phi}{\log \phi}$$

Z = rounded off in multiples of 2 & 3.

R_5 Series — $\sqrt[5]{10} = 1.58$

R_{10} series — $\sqrt[10]{10} = 1.26$

R_{20} Series — $\sqrt[20]{10} = 1.12$

R_{40} series — $\sqrt[40]{10} = 1.06$

R_{80} series — $\sqrt[80]{10} = 1.03$

Structural Diagrams & their Analysis to Select Best possible version.

Suppose a speed on one shaft yield two speed values on the next shaft, i.e. no of speed steps of the particular transmission group is $p = 2$, the two new speed should be in the following range.

$$i_{max} = 2 \qquad i_{min} = \frac{1}{4}$$

The transmission range for the group

$$i_g = \frac{i_{max}}{i_{min}} = 8$$

Suppose there are z steps $n_1, n_2 \ldots n_z$ on a particular transmission group such that

$$\frac{n_2}{n_1} = \frac{n_3}{n_2} = \ldots \frac{n_z}{n_{z-1}} = k.$$

$$k = \phi^{x}.$$

where x is known as characteristic of the transmission group & denotes the no. of steps of the spindle rpm geometric progression by which two adjacent rpm value of the particular group are seperated.

The transmission range of m^{th} group can be calculated

$$i_m = \phi^{(P_m - 1) X_m}.$$

for obtaining particular number of speed steps using min. no of gears, its necessary that

$P_1 = P_2 \;---\; = p_v \;//\; p = z^{1/v}$.

The number of speed steps is represented by.

$z = P_1 \times P_2 \times \dots P_v$. // P = no of speed steps.

More elaborate expression.

$Z_v = P_1(x_1)\ P_2(x_2)\ P_3(x_3) \dots P_v(x_v)$.

where $x_1 = 1$ $x_2 = P_1$ $x_3 = P_1 P_2 \dots$

Eg: Let no of speed steps $z = 12$. & Let it be realized in 3 stages $v = 3$.

The number 12 can be written as multiplicatio of 2 & 3 in 6 different ways. One such combination. // here. 2 groups are common ie $p = 2$, $q = 2$ ∴ no of posi $= \frac{3!}{1!} = 6$.

p_1 p_2 p_3.

$z = 2 \times 3 \times 2$.

we can have six structural formula.

P_1 x_1 $P_2\ x_2$ $x_3.P_3$.

a). $\Rightarrow$ $z = P_1(x_1)\ P_2 x_2\ P_3(x_3)$.

$x_1 = 1$ $x_2 = P_1 = 2$ $x_3 = P_1 P_2$:

$x_1 = 1$ $x_2 = 2$ $x_3 = 6$.

$x_1 = 1$

$$\boxed{Z = 2(1)\ 3(2)\ 2(6)}$$

shaft 2 shaft 3 shaft 4.

ϕ^m = group q $= L_m = \phi^{(P_m - 1) x_m}$

$2(1) \rightarrow$ shaft 2 has 2 gears & each yield 1 speed ray.

$3(2) \rightarrow$ shaft 3 has 3 gear & each yields 2 speed ray.

$2(6) \rightarrow$ shaft 4 has 2 gears & each yields 6 speed rays.

b). P_1 x_3 $P_2\ x_1$ $x_2\ P_3$.

$z = P_1(x_3)\ P_2(x_1)\ P_3(x_2)$.

where $x_1 = 1$, $x_2 = P_2 = 3$, $x_3 = P_2 P_3 = 6$.

$$\boxed{z = 2(6)\ 3(1)\ 2(3)}$$

c). P_1 x_2 $P_2 x_3$. $x_1 P_3$.

$z = P_1(x_2)\ P_2(x_3)\ P_3(x_1)$.

$x_1 = 1$ $x_2 = P_3 = 2$ $x_3 = P_1 P_3 = 4$.

$z = 2(2)\ 3(4)\ 2(1)$

) $Z = 2 \times 3 \times 2.$

Next form: P_1 x_2 — $P_2 x_1$ — $x_3 P_3$.

$Z = P_1(x_2) \quad P_2(x_1) \quad P_3(x_3)$

$x_1 = 1 \quad x_2 = P_2 \quad x_3 = P_1 P_2.$

$x_1 = 1 \quad x_2 = 3 \quad x_3 = 6$

$$\boxed{Z = 2(3) \quad 3(1) \quad 2(6)}$$

).

$P_1 (x_1)$ — $P_2(x_3)$ — $(x_2) P_3$

$Z = P_1(x_1) \quad P_2(x_3) \quad P_3(x_2).$

$x_1 = 1 \quad x_2 = P_1 = 2, \quad x_3 = 4 = (P_1 P_3)$

$$\boxed{2(1) \quad 3(4) \quad 2(2).}$$

) $P_1 (x_3)$ — $P_2(x_2)$ — $(x_1) P_3$.

$Z = P_1(x_3) \quad P_2(x_2) \quad P_3(x_1).$

$x_1 = 1 \quad x_2 = P_3 \quad x_3 = P_2 P_3.$

$x_1 = 1 \quad x_2 = 2 \quad x_3 = 6.$

$$\boxed{Z = 2(6) \quad 3(2) \quad 2(1)}$$

$\therefore$ The six structural formulae are.

1) $Z = 2(1)\ 3(2)\ 2(6)$

2) $Z = 2(6)\ 3(1)\ 2(3)$

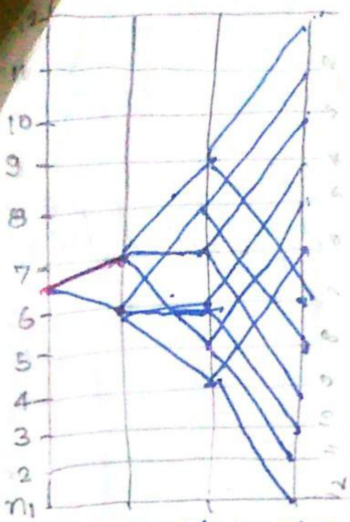
3) $Z = 2(2)\ 3(4)\ 2(1)$ -

4) $Z = 2(3)\ 3(1)\ 2(6).$

5). $Z = 2(1)\ 3(4)\ 2(2).$

6) $Z = 2(6)\ 3(2)\ 2(1).$

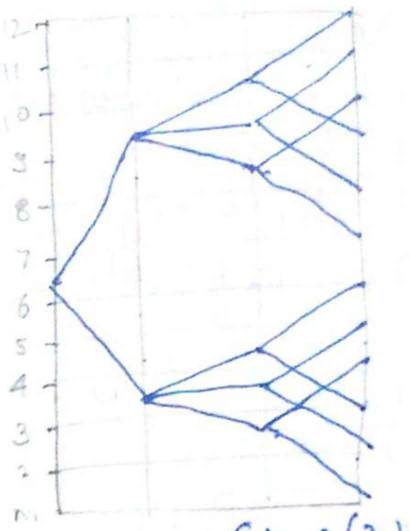
$x_1 = 1$ [Main transmission group]



a 2(1) 3(2) 2(6).

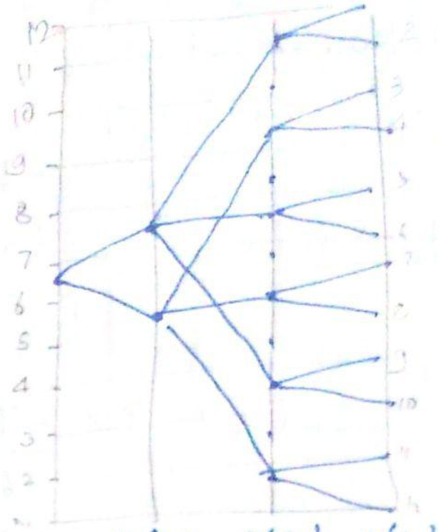
2 gears with spacing of 1

7 + 9 + 12 = 28



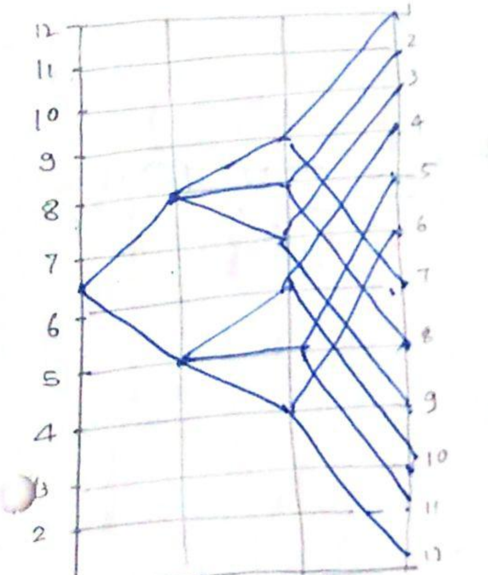
b) 2(6) 3(1) 2(3)

3.5 + 9.5 + 10.5 + 12 = 32



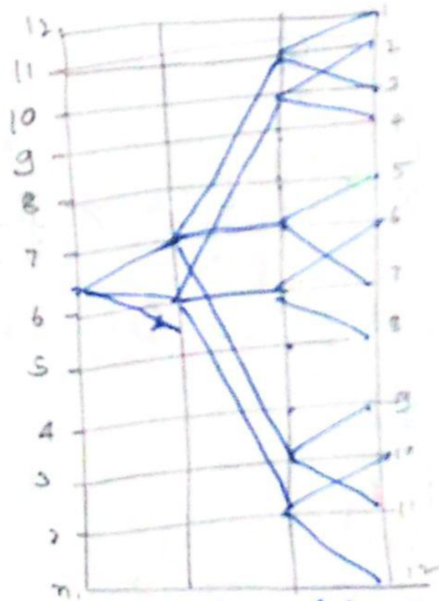
2(2) 3(4) 2(1).

7.5 + 11.5 + 12 = 31



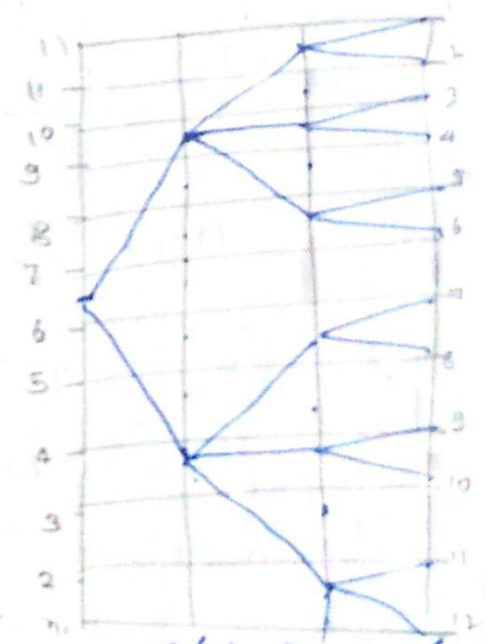
d 2(3) 3(1) 2(6)

8 + 9 + 12 = 29



e. 2(1) 3(4) 2(2)

7 + 11 + 12 = 30



2(6) 3(2) 2(1).

9.5 + 11.5 + 12 = 33

The structural diagram gives information about the speed box

- no of shafts in the speed box
- no of gears on each shaft.
- order of changing transmissions in individual groups to get the desired spindle speed
- Transmission range & characteristic of each group.

The selection of the best version is guided by following two factors.

A) Transmission ratio restriction
$i_g \quad i_g \leq 8$.

B) Minimum total shaft size.

If rpm for GP then torque also will be in GP.

we have $P = \frac{2\pi n M_t}{60 \times 10^3}$ kw.

$$M_t = \left[\frac{60 \times 10^3}{2\pi n}\right] \frac{P}{n}.$$

$$\boxed{M_t = \frac{kP}{n}} \quad -①.$$

we have $\frac{M_t}{J} = \frac{\tau}{r}$.

$$M_t = \frac{\tau}{r} \times J = \frac{\tau \times 2}{d} \times \frac{\pi}{32} d^4 = \frac{\tau \pi}{16} d^3.$$

$$\boxed{M_t = 0.2\, d^3 \tau} \quad - ②.$$

from ① & ②.

$$0.2 d^3 \tau = \frac{kP}{n}$$

$$d = \left[\frac{k}{0.2\tau} \frac{P}{n}\right]^{1/3}.$$

$$\therefore d_1 = \left[\frac{k}{0.2\tau} \frac{P}{n_1}\right]^{1/3} \qquad d_2 = \left[\frac{k}{0.2\tau} \frac{P}{n_2}\right]^{1/3}.$$

$$\therefore \frac{d_1}{d_2} = \left(\frac{n_2}{n_1}\right)^{1/3} = \phi^{1/3}.$$

$\therefore$ Shaft dia also constitute a GP.

The best version which ensures.

$$\sum_{i=1}^{u+1} d = \text{min}.$$